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Analysis of realizable rational decimated nonuniform filter banks with direct structure

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Abstract

In the design of nonuniform filter banks (NUFBs) with direct structure, the location of each analysis filter, corresponding to the sampling factor satisfying maximal decimation condition, should be set properly to avoid large aliasing. In this paper, a necessary and sufficient condition for the setting of the location of each analysis filter is derived. The NUFBs, we focus on, have rational decimation factors. Based on the derived condition, the frequency support of each analysis filter for the realizable NUFBs can be determined directly in such a way that the analysis filters can extract the corresponding bands of the input signal. This provides a guideline for the design of NUFBs with direct structure in choosing proper locations of analysis filters.

Keywords: Nonuniform filter banks; Rational sampling factors; Large aliasing; Locations of analysis filters

1. Introduction

Nonuniform filter banks (NUFBs) are extensively studied due to their flexibility in frequency partitioning [1,2]. In general, there are two structures for the NUFB design: indirect structure [3–5] and direct structure [6–10]. Indirect structure is such a structure that certain channels of a uniform filter bank are merged together using the synthesis filters of transmultiplexers with smaller number of channels [3]. This method may lead to a long system delay due to its two-stage architecture. In addition, although the perfect-reconstruction property can be structurally imposed as long as the uniform filter bank and the recombination transmultiplexers are of perfect-reconstruction, the equivalent analysis filters are generally not linear time-invariant (LTI), which makes it difficult to perform the optimization, leading to the suboptimality in filter quality. In contrast, the direct structure as shown in Fig. 1 is more attractive because it is a one-stage structure that can result in a low system delay. Furthermore, it allows one to have a complete control over the desired frequency characteristics of the LTI analysis filters [11]. It should be pointed out that for the NUFBs applied in direct structure, each analysis filter should satisfy certain requirements. Otherwise, even with ideal filters, the NUFBs cannot be realized due to large aliasing appearing in the passband region of analysis filters.

For the problem of large aliasing, Li et al. [9] addressed the issue of integer decimated NUFBs. If the cut-off frequencies of each analysis filter are the integer multiple of its bandwidth, large aliasing can be avoided, and furthermore the locations of analysis filters will be obtained directly based on the frequency partitioning scheme of the input signal since there is no upsampler before analysis filters. However, unlike the integer decimated filter banks, in the design of rational decimated NUFBs, the problem of large aliasing and the selection of analysis filters become more complicated because the input signal is interpolated before passing through the analysis filters. Studies in Refs.

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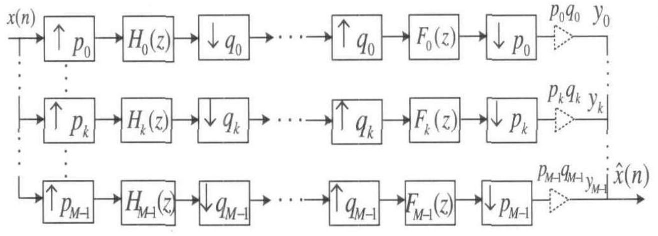

Fig. 1. Direct structure of NUFBs.

[6–8] focused on the effective design methods for realizable NUFBs with rational decimation factors, but there was no discussion on avoiding large aliasing, the realizable NUFBs means that they can be realized in the near-perfect-reconstruction sense. In Ref. [10], the constraint on avoiding large aliasing has been carried out for the rational decimated NUFBs, which is the same as that in Ref. [9]. Kovacevic et al. [11], from another viewpoint, gave a requirement for the qth band filter H whose spectrum has to be located in $[\frac{s}{q}\pi, \frac{s+1}{q}\pi]$, where $s \in \{0,1,\ldots,q-1\}$, to avoid large aliasing. This requirement can determine whether large aliasing will appear or not in each channel. However, it cannot be used to derive the frequency support of each analysis filter. Thus, in the design of rational decimated NUFBs with direct structure, it is desirable that we cannot only determine whether large aliasing appears, but also directly obtain the frequency support of each analysis filter.

In this study, we consider the selection of frequency supports of analysis filters in rational decimated NUFB systems. Starting with the possible frequency locations of analysis filters, we derive a necessary condition that each analysis filter must satisfy to extract the input signal correctly. Furthermore, in order to realize the desired spectrum splitting of the input signal, large aliasing must be avoided in NUFB systems. Subsequently, a requirement of avoiding large aliasing for a filter followed by a downsampler is given. Based on the necessary condition and the requirement, a necessary and sufficient condition is derived. This condition can be used to determine whether a rational decimated NUFB is realizable, and derive the valid location of each analysis filter for the realizable NUFBs. Note that because synthesis filters are the reverse operation of analysis filters, they have the same frequency supports as those of analysis filters. Thus, once the locations of analysis filters are obtained, those of synthesis filters will be known accordingly. For this reason, only the locations of analysis filters will be discussed in this study.

In what follows, all analysis and synthesis filters have real coefficients, and only the frequency region of $[0,\pi]$ is considered.

2. Possible locations of analysis filters

2.1. Direct structure

Fig. 1 shows the typical direct structure for the design of M-channel NUFBs with rational sampling factors, where $H_k(z)$ and $F_k(z)$ denote the analysis and the synthesis filters, respectively, and p_k/q_k are the rational sampling factors $(k = 0,1,\ldots,M-1)$. For critical sampling, they satisfy the condition of $\sum_{k=0}^{M-1} p_k/q_k = 1$. In this work, we only consider the critical case. The direct structure shown in Fig. 1 performs a spectrum partitioning of the input signal as given in Fig. 2. For Band k, it contains the input frequency with the range over $[\sum_{i=0}^{k-1}\frac{p_i}{q_i}\pi, \sum_{i=0}^{k}\frac{p_i}{q_i}\pi]$ $(k = 0,1,\ldots,M-1)$, where the sum is defined as 0 if the upper bound is negative. In order to extract each band correctly, the analysis filters should be in proper locations.

For the integer decimated NUFB systems, where $p_k = 1$ $(k = 0,1,\ldots,M-1)$, the frequency location of each analysis filter can be obtained immediately based on the frequency partitioning scheme of the input signal. That is to say, the frequency support of each analysis filter is the same as that of its corresponding band to be extracted. However, unlike the integer decimated NUFB systems, in the design of rational decimated NUFBs, the locations of analysis filters should be treated carefully, because the input signal is interpolated by p_k $(k = 0,1,\ldots,M-1)$ before passing through the analysis filters. More specifically, the input signal $X(z)$, after interpolated by p_k for the kth channel, $k \in \{0,1,\ldots,M-1\}$, becomes $X(z^{p_k})$ whose spectrum is shown in Fig. 3. Since $X(z^{p_k})$ has the period of $2\pi/p_k$, and each period has the same information as that of $X(z)$, $H_k(z)$ should be situated at proper locations to extract the same information as that of Band k when $X(z^{p_k})$ passes through it. In what follows, we will discuss the locations of analysis filters of rational decimated NUFBs. Due to the different properties, the lowpass, bandpass and highpass analysis filters are discussed separately.

2.2. Analysis of lowpass, bandpass and highpass analysis filters

Considering the first channel shown in Fig. 1, the input signal is interpolated by p_0 before passing through the lowpass filter $H_0(z)$. Consequently, to extract Band 0 of the input frequency, $H_0(z)$ can be located in $[0,\pi/q_0]$. For a better understanding, the NUFB with sampling factors [3/11, 5/11, 2/11, 1/11] denoted as Example 1 is given. Its partitioning scheme is shown in Fig. 4(a). Since the first channel has the sampling factor 3/11, the lowpass filter $H_0(z)$ has to be located in $[0,\pi/11]$ to extract Band 0 as shown in Fig. 4(a). It is similar to that of another NUFB with sampling factors [2/9, 1/3, 4/9] denoted as Example 2,

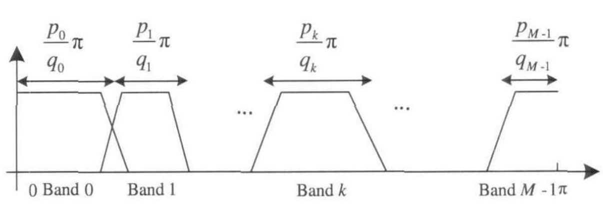

Fig. 2. The desired spectral splitting of input signal.

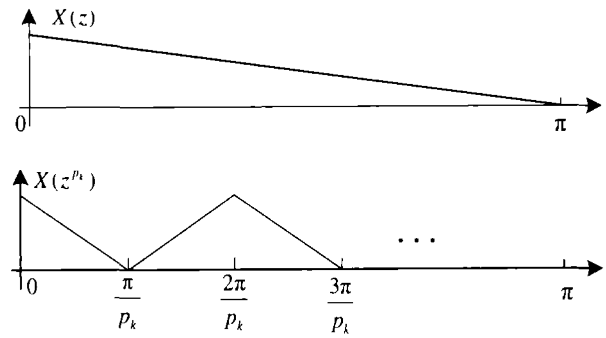

Fig. 3. Spectrum of the input signal $X(z)$ and $X(z^{p_k})$.

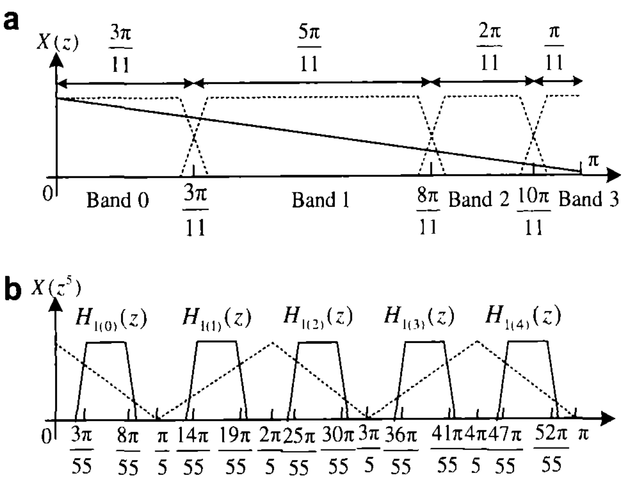

Fig. 4. A 4-channel NUFB with sampling factors [3/11, 5/11, 2/11, 1/11]. (a) Spectrum splitting of the input signal $X(z)$; (b) spectrums of $X(z^5)$ and $Hk(z)$.

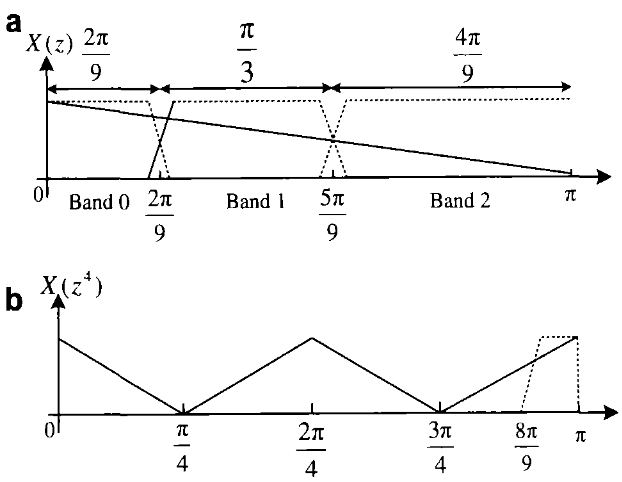

Fig. 5. A 3-channel NUFB with the sampling factors [2/9, 1/3, 4/9]. (a) Spectrum splitting of the input signal $X(z)$; (b) spectrums of $X(z^4)$.

whose spectrum partitioning is sketched in Fig. 5(a). $H_0(z)$ is located in $[0, \pi/9]$ .The bandpass filter, $H_k(z)$ $(k = 1, 2, \ldots, M-2)$, following the upsampler p_k, may have p_k locations and each location should be in the region of $[\frac{d_k}{p_k}\pi, \frac{d_k+1}{p_k}\pi]$, where $d_k = 0, \ldots, p_k - 1$, for $X(z^{p_k})$ has the period of $2\pi/p_k$ and each period contains the same frequency information as $X(z)$ (shown in Fig. 3). The p_k possible locations of $H_k(z)$ are shown in Table 1, where $H_{k(i)}(z)$ denotes the ith possible choice of $H_k(z)$. Based on Table 1, we can formulate p_k locations of $H_k(z)$ as

$$\left[\frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}+d_k\right), \frac{\pi}{p_k}\left(\sum_{i=0}^{k}\frac{p_i}{q_i}+d_k\right)\right], \quad d_k \text{ is even} \tag{1}$$

$$\left[\frac{\pi}{p_k}\left(d_k+1-\sum_{i=0}^{k}\frac{p_i}{q_i}\right), \frac{\pi}{p_k}\left(d_k+1-\sum_{i=0}^{k-1}\frac{p_i}{q_i}\right)\right], \quad d_k \text{ is odd} \tag{2}$$

where d_k=0,...,$p_k - 1$ $(k = 1, 2, \ldots, M-2)$. To extract Band k of the input spectrum, $H_k(z)$ has to be situated at one of the above p_k locations. This is a necessary condition for $H_k(z)$ to extract its corresponding band.

Considering the second channel of Example 1, $X(z)$, after interpolated by 5, becomes $X(z^5)$, which is shown in Fig. 4(b) (dashed line). Consequently, the bandpass filter $H_1(z)$ has five possible locations. From (1) and (2), we obtain the five locations, which are $[3\pi/55, 8\pi/55]$ with $d_1 = 0$, $[14\pi/55, 19\pi/55]$ with $d_1 = 1$, $[25\pi/55, 30\pi/55]$ with $d_1 = 2$, $[36\pi/55, 41\pi/55]$ with $d_1 = 3$, and $[47\pi/55, 52\pi/55]$ with $d_1 = 4$, shown as the real line in Fig. 4(b). Similarly, in the third channel, we obtain the two possible locations of the bandpass filter $H_2(z)$ which are $[4\pi/11, 5\pi/11]$ with $d_2 = 0$ and $[6\pi/11, 7\pi/11]$ with $d_2 = 1$. For the second channel of Example 2, the bandpass filter $H_1(z)$ has only one location $[2\pi/9, 5\pi/9]$ since $p_1 = 1$. To extract the bands of the input frequency correctly, bandpass filters have to be in one of their corresponding possible locations.

The highpass filter $H_{M-1}(z)$, which will extract Band $M-1$, only exists if p_{M-1} is odd, because the low frequency of the input signal will appear in the region around π after interpolated by even p_{M-1}. When p_{M-1} is odd, $H_{M-1}(z)$ is located in $[\pi - \pi/q_0, \pi]$. For the highpass channel in Example 1, the spectrum location of the highpass filter is in $[10\pi/11, \pi]$. However, the highpass filter does not exist in Example 2 because the region $[8\pi/9, \pi]$ of $X(z^4)$ (dashed regions in Fig. 5(b)) does not contain the same frequency as that of Band 2 (Fig. 5(a)), but contains the low frequency information of $X(z)$.

3. Analysis of the realization of NUFBs

For a rational decimated NUFB, each analysis filter should be located in one of its possible locations. This is a necessary condition for analysis filters to perform the desired spectrum splitting scheme of the input signal. However, the necessary condition is not enough. Not all possible locations are valid for being chosen as frequency supports because of the large aliasing caused by downsamplers. For a realizable NUFB, each chosen location should meet certain requirement for avoiding large aliasing that cannot be eliminated even with ideal filters. Otherwise, the NUFB cannot be realized. Kovacevic et al. [11] addressed such requirement as given in the following proposition.

Table 1
Possible frequency locations of $H_k(z)$ in region $[0, \pi]$

$H_k(z)$	Left cut-off frequency	Right cut-off frequency	In the region
$H_{k(0)}(z)$	$\frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}\right)$	$\frac{\pi}{p_k}\left(\sum_{i=0}^{k}\frac{p_i}{q_i}\right)$	$\left[0, \frac{\pi}{p_k}\right](d_k = 0)$
$H_{k(1)}(z)$	$\frac{2\pi}{p_k} - \frac{\pi}{p_k}\left(\sum_{i=0}^{k}\frac{p_i}{q_i}\right)$	$\frac{2\pi}{p_k} - \frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}\right)$	$\left[\frac{\pi}{p_k}, \frac{2\pi}{p_k}\right](d_k = 1)$
$H_{k(2)}(z)$	$\frac{2\pi}{p_k} + \frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}\right)$	$\frac{2\pi}{p_k} + \frac{\pi}{p_k}\left(\sum_{i=0}^{k}\frac{p_i}{q_i}\right)$	$\left[\frac{2\pi}{p_k}, \frac{3\pi}{p_k}\right](d_k = 2)$
$H_{k(3)}(z)$	$\frac{4\pi}{p_k} - \frac{\pi}{p_k}\left(\sum_{i=0}^{k}\frac{p_i}{q_i}\right)$	$\frac{4\pi}{p_k} - \frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}\right)$	$\left[\frac{3\pi}{p_k}, \frac{4\pi}{p_k}\right](d_k = 3)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Proposition. *The qth band filter H with real coefficients, followed by the downsampler q, meant to avoid large aliasing, has to be located in*

$$\left[\frac{n}{q}\pi, \frac{n+1}{q}\pi\right] \tag{3}$$

for certain $n \in \{0, 1, \ldots, q-1\}$.

The expression (3) is only a constraint for a filter to avoid large aliasing and cannot be used to obtain the valid location of each analysis filter. However, combining the necessary condition (1) and (2) in Section 2 and the constraint (3), we cannot only find the possible locations of analysis filters easily, but also directly determine which location is valid so as to avoid large aliasing.

It is apparent that the lowpass filter located in $[0, \pi/q_0]$ satisfies (3). The same conclusion can be reached for the highpass filter which is located in $[\pi - \pi/q_{M-1}, \pi]$ when p_{M-1} is odd. The highpass filter does not exist in the case of p_{M-1} being even. For the bandpass filter, $H_k(z)$ $(k = 1, 2, \ldots, M-2)$, large aliasing caused by its following decimator can be avoided, if at least one of its p_k possible locations, derived from (1) and (2), satisfies the requirement of (3).

Consequently, the following result holds.

Theorem. *For a critical decimated M-channel NUFB with rational sampling factors* $[p_0/q_0, p_1/q_1, \ldots, p_{M-1}/q_{M-1}]$ *if* there exists at least one possible location of each bandpass filter satisfying

$$\frac{\pi}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i} + d_k\right) = \frac{n_k}{q_k}\pi \quad \text{or} \quad \frac{q_k}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i} + d_k\right) = n_k \tag{4}$$

where d_k *is even or*

$$\frac{\pi}{p_k}\left(d_k + 1 - \sum_{i=0}^{k}\frac{p_i}{q_i}\right) = \frac{n_k}{q_k}\pi \quad \text{or} \quad \frac{q_k}{p_k}\left(d_k + 1 - \sum_{i=0}^{k}\frac{p_i}{q_i}\right) = n_k \tag{5}$$

where d_k *is odd for some* $n_k \in \{0, 1, \ldots, q_k - 1\}$, $d_k = 0, 1, \ldots, p_k - 1$, and p_{M-1} *is odd, the NUFB system can be realized with direct structure by using proper design methods.*

This is a necessary and sufficient condition on the realization of critical decimated NUFBs with rational sampling factors. Thus, in the design of realizable NUFBs, the valid locations of analysis filters can be derived directly. This is also true for synthesis filters since they have the same frequency supports as those of analysis filters.

Referring to Example 1, both the lowpass filter $H_0(z)$ with the frequency support of $[0, \pi/11]$ and the highpass filter $H_3(z)$ with the frequency support of $[10\pi/11, \pi]$ satisfy the requirement of (3); for the five possible locations of $H_1(z)$ shown in Fig. 4(b), the location $[25\pi/55, 30\pi/55]$ $([5\pi/11, 6\pi/11])$ with $d_1 = 2$ meets the requirement of (4); the two possible locations of $H_2(z)$, $[4\pi/11, 5\pi/11]$ and $[6\pi/11, 7\pi/11]$, satisfy (4) and (5), respectively. Thus, Example 1 can be realized with direct structure by using proper design methods. However, for Example 2, since the bandpass filter with the frequency support of $[2\pi/9, 5\pi/9]$ does not satisfy (4), (5), and the highpass filter does not exist, it cannot be realized even with ideal analysis filters because of the large aliasing effect.

4. Conclusion

In this study, we have discussed the possible locations of analysis filters in the direct structure, and derived the necessary condition for each analysis filter to extract the corresponding band of the input signal. We have also given a requirement so as to avoid large aliasing. Finally, the necessary and sufficient condition on the realizable NUFBs has been derived. By using this condition, one can easily determine whether a rational decimated NUFB can be realized. For the realizable NUFB case, the valid location of each analysis/synthesis filter can be readily derived.

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